

PLTW Engineering Formula Sheet 2014

1.0 Statistics

Mean

$$\mu = \frac{\sum x_i}{N} \quad (1.1a) \qquad \bar{x} = \frac{\sum x_i}{n} \quad (1.1b)$$

μ = population mean
 \bar{x} = sample mean
 $\sum x_i$ = sum of all data values (x_1, x_2, x_3, \dots)
 N = size of population
 n = size of sample

Median

Place data in ascending order.
 If N is odd, median = central value (1.2)
 If N is even, median = mean of two central values
 N = size of population

Range (1.5)

$$\text{Range} = x_{\max} - x_{\min} \quad (1.3)$$

x_{\max} = maximum data value
 x_{\min} = minimum data value

Mode

Place data in ascending order.
 Mode = most frequently occurring value (1.4)

If two values occur with maximum frequency the data set is *bimodal*.
 If three or more values occur with maximum frequency the data set is *multi-modal*.

Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} \quad (\text{Population}) \quad (1.5a)$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \quad (\text{Sample}) \quad (1.5b)$$

σ = population standard deviation
 s = sample standard deviation
 x_i = individual data value (x_1, x_2, x_3, \dots)
 μ = population mean
 \bar{x} = sample mean
 N = size of population
 n = size of sample

2.0 Probability

Frequency

$$f_x = \frac{n_x}{n} \quad (2.1)$$

f_x = relative frequency of outcome x
 n_x = number of events with outcome x
 n = total number of events

Binomial Probability (order doesn't matter)

$$P_k = \frac{n!(p^k)(q^{n-k})}{k!(n-k)!} \quad (2.2)$$

P_k = binomial probability of k successes in n trials
 p = probability of a success
 $q = 1 - p$ = probability of failure
 k = number of successes
 n = number of trials

Independent Events

$$P(A \text{ and } B \text{ and } C) = P_A P_B P_C \quad (2.3)$$

$P(A \text{ and } B \text{ and } C)$ = probability of independent events A and B and C occurring in sequence
 P_A = probability of event A

Mutually Exclusive Events

$$P(A \text{ or } B) = P_A + P_B \quad (2.4)$$

$P(A \text{ or } B)$ = probability of either mutually exclusive event A or B occurring in a trial
 P_A = probability of event A

Conditional Probability

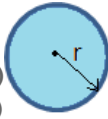
$$P(A|D) = \frac{P(A) \cdot P(D|A)}{P(A) \cdot P(D|A) + P(\sim A) \cdot P(D|\sim A)} \quad (2.5)$$

$P(A|D)$ = probability of event A given event D
 $P(A)$ = probability of event A occurring
 $P(\sim A)$ = probability of event A not occurring
 $P(D|\sim A)$ = probability of event D given event A did not occur

3.0 Plane Geometry

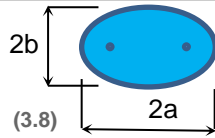
Circle

Circumference = $2\pi r$ (3.1)
 Area = πr^2 (3.2)



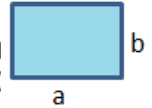
Ellipse

Area = $\pi a b$ (3.8)



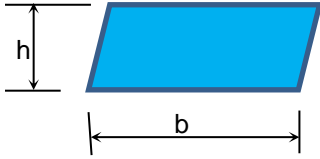
Rectangle

Perimeter = $2a + 2b$ (3.9)
 Area = ab (3.10)



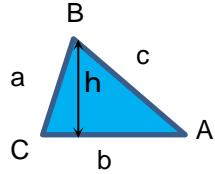
Parallelogram

Area = bh (3.3)



Triangle (3.6)

Area = $\frac{1}{2}bh$ (3.11)
 $a^2 = b^2 + c^2 - 2bc \cdot \cos \angle A$ (3.12)
 $b^2 = a^2 + c^2 - 2ac \cdot \cos \angle B$ (3.13)
 $c^2 = a^2 + b^2 - 2ab \cdot \cos \angle C$ (3.14)



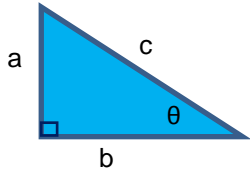
Right Triangle

$c^2 = a^2 + b^2$ (3.4)

$\sin \theta = \frac{a}{c}$ (3.5)

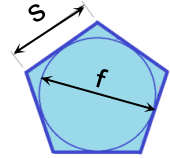
$\cos \theta = \frac{b}{c}$ (3.6)

$\tan \theta = \frac{a}{b}$ (3.7)



Regular Polygons

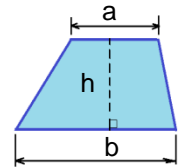
Area = $n \frac{s(\frac{1}{2}f)}{2} = \frac{ns^2}{4 \tan(\frac{180}{n})}$ (3.15)



n = number of sides

Trapezoid

Area = $\frac{1}{2}(a + b)h$ (3.16)

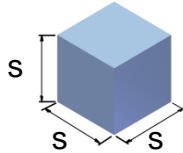


4.0 Solid Geometry

Cube

Volume = s^3 (4.1)

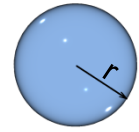
Surface Area = $6s^2$ (4.2)



Sphere

Volume = $\frac{4}{3}\pi r^3$ (4.8)

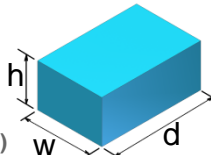
Surface Area = $4\pi r^2$ (4.9)



Rectangular Prism

Volume = $w dh$ (4.3)

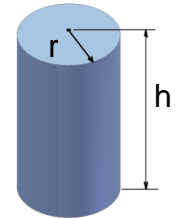
Surface Area = $2(wd + wh + dh)$ (4.4)



Cylinder

Volume = $\pi r^2 h$ (4.10)

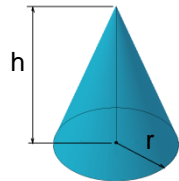
Surface Area = $2\pi r h + 2\pi r^2$ (4.11)



Right Circular Cone

Volume = $\frac{\pi r^2 h}{3}$ (4.5)

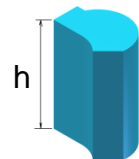
Surface Area = $\pi r \sqrt{r^2 + h^2}$ (4.6)



Irregular Prism

Volume = Ah (4.12)

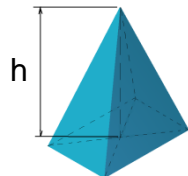
A = area of base



Pyramid

Volume = $\frac{Ah}{3}$ (4.7)

A = area of base



5.0 Constants

$g = 9.8 \text{ m/s}^2 = 32.27 \text{ ft/s}^2$

$G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$

$\pi = 3.14159$

6.0 Conversions

Mass/Weight (6.1)

1 kg	= 2.205 lb _m
1 slug	= 32.2 lb _m
1 ton	= 2000 lb
1 lb	= 16 oz

Area (6.4)

1 acre	= 4047 m ²
	= 43,560 ft ²
	= 0.00156 mi ²

Force (6.7)

1 N	= 0.225 lb
1 kip	= 1,000 lb

Energy (6.10)

1 J	= 0.239 cal
	= 9.48 x 10 ⁻⁴ Btu
	= 0.7376 ft·lb _f
1 kW h	= 3,600,000 J

Length (6.2)

1 m	= 3.28 ft
1 km	= 0.621 mi
1 in.	= 2.54 cm
1 mi	= 5280 ft
1 yd	= 3 ft

Volume (6.5)

1L	= 0.264 gal
	= 0.0353 ft ³
	= 33.8 fl oz
1mL	= 1 cm ³ = 1 cc

Pressure (6.8)

1 atm	= 1.01325 bar
	= 33.9 ft H ₂ O
	= 29.92 in. Hg
	= 760 mm Hg
	= 101,325 Pa
	= 14.7 psi
1psi	= 2.31 ft of H ₂ O

7.0 Defined Units

1 J	= 1 N·m
1 N	= 1 kg·m / s ²
1 Pa	= 1 N / m ²
1 V	= 1 W / A
1 W	= 1 J / s
1 Ω	= 1 V / A
1 Hz	= 1 s ⁻¹
1 F	= 1 A·s / V
1 H	= 1 V·s / V

Time (6.3)

1 d	= 24 h
1 h	= 60 min
1 min	= 60 s
1 yr	= 365 d

Temperature Unit Equivalents (6.6)

1 K	= 1 °C
	= 1.8 °F
	= 1.8 °R

See below for temperature calculation

Power (6.9)

1 W	= 3.412 Btu/h
	= 0.00134 hp
	= 14.34 cal/min
	= 0.7376 ft·lb _f /s
1 hp	= 550 ft·lb/sec

8.0 SI Prefixes

Numbers Less Than One

Power of 10	Prefix	Abbreviation
10 ⁻¹	deci-	d
10 ⁻²	centi-	c
10 ⁻³	milli-	m
10 ⁻⁶	micro-	μ
10 ⁻⁹	nano-	n
10 ⁻¹²	pico-	p
10 ⁻¹⁵	femto-	f
10 ⁻¹⁸	atto-	a
10 ⁻²¹	zepto-	z
10 ⁻²⁴	yocto-	y

Numbers Greater Than One

Power of 10	Prefix	Abbreviation
10 ¹	deca-	da
10 ²	hecto-	h
10 ³	kilo-	k
10 ⁶	Mega-	M
10 ⁹	Giga-	G
10 ¹²	Tera-	T
10 ¹⁵	Peta-	P
10 ¹⁸	Exa-	E
10 ²¹	Zetta-	Z
10 ²⁴	Yotta-	Y

9.0 Equations

Mass and Weight

$$m = VD_m \quad (9.1)$$

$$W = mg \quad (9.2)$$

$$W = VD_w \quad (9.3)$$

V = volume

D_m = mass density

m = mass

D_w = weight density

W = weight

g = acceleration due to gravity

Temperature

$$T_K = T_C + 273 \quad (9.4)$$

$$T_R = T_F + 460 \quad (9.5)$$

$$T_F = \frac{9}{5} T_C + 32 \quad (9.6)$$

T_K = temperature in Kelvin

T_C = temperature in Celsius

T_R = temperature in Rankin

T_F = temperature in Fahrenheit

Force and Moment

$$F = ma \quad (9.7a) \quad M = Fd_{\perp} \quad (9.7b)$$

F = force

m = mass

a = acceleration

M = moment

d_⊥ = perpendicular distance

Equations of Static Equilibrium

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_P = 0 \quad (9.8)$$

F_x = force in the x-direction

F_y = force in the y-direction

M_P = moment about point P

9.0 Equations (Continued)

Energy: Work

$$W = F_{\parallel} \cdot d \quad (9.9)$$

W = work
 F_{\parallel} = force parallel to direction of displacement
d = displacement

Power

$$P = \frac{E}{t} = \frac{W}{t} \quad (9.10)$$

$$P = \tau \omega \quad (9.11)$$

P = power
E = energy
W = work
t = time
 τ = torque
 ω = angular velocity

Efficiency

$$\text{Efficiency (\%)} = \frac{P_{\text{out}}}{P_{\text{in}}} \cdot 100\% \quad (9.12)$$

P_{out} = useful power output
 P_{in} = total power input

Energy: Potential

$$U = mgh \quad (9.13)$$

U = potential energy
m = mass
g = acceleration due to gravity
h = height

Energy: Kinetic

$$K = \frac{1}{2} mv^2 \quad (9.14)$$

K = kinetic energy
m = mass
v = velocity

Energy: Thermal

$$\Delta Q = mc\Delta T \quad (9.15)$$

ΔQ = change in thermal energy
m = mass
c = specific heat
 ΔT = change in temperature

Fluid Mechanics

$$p = \frac{F}{A} \quad (9.16)$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (\text{Charles' Law}) \quad (9.17)$$

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad (\text{Gay-Lussanc's Law}) \quad (9.18)$$

$$p_1 V_1 = p_2 V_2 \quad (\text{Boyle's Law}) \quad (9.19)$$

$$Q = Av \quad (9.20)$$

$$A_1 v_1 = A_2 v_2 \quad (9.21)$$

$$P = Qp \quad (9.22)$$

absolute pressure = gauge pressure + atmospheric pressure (9.23)

p = absolute pressure
F = force
A = area
V = volume
T = absolute temperature
Q = flow rate
v = flow velocity
P = power

Mechanics

$$\bar{s} = \frac{d}{t} \quad (9.24)$$

$$\bar{v} = \frac{\Delta d}{\Delta t} \quad (9.25)$$

$$a = \frac{v_f - v_i}{t} \quad (9.26)$$

$$X = \frac{v_i^2 \sin(2\theta)}{-g} \quad (9.27)$$

$$v = v_i + at \quad (9.28)$$

$$d = d_i + v_i t + \frac{1}{2} at^2 \quad (9.29)$$

$$v^2 = v_i^2 + 2a(d - d_i) \quad (9.30)$$

$$\tau = dF \sin \theta \quad (9.31)$$

\bar{s} = average speed
 \bar{v} = average velocity
v = velocity
 v_i = initial velocity (t=0)
a = acceleration
X = range
t = time
 Δd = change in displacement
d = distance
 d_i = initial distance (t=0)
g = acceleration due to gravity
 θ = angle
 τ = torque
F = force

Electricity

Ohm's Law

$$V = IR \quad (9.32)$$

$$P = IV \quad (9.33)$$

$$R_T (\text{series}) = R_1 + R_2 + \dots + R_n \quad (9.34)$$

$$R_T (\text{parallel}) = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}} \quad (9.35)$$

Kirchhoff's Current Law

$$I_T = I_1 + I_2 + \dots + I_n$$

or $I_T = \sum_{k=1}^n I_k \quad (9.36)$

Kirchhoff's Voltage Law

$$V_T = V_1 + V_2 + \dots + V_n$$

or $V_T = \sum_{k=1}^n V_k \quad (9.37)$

V = voltage
 V_T = total voltage
I = current
 I_T = total current
R = resistance
 R_T = total resistance
P = power

Thermodynamics

$$P = Q' = AU\Delta T \quad (9.38)$$

$$P = Q' = \frac{\Delta Q}{\Delta t} \quad (9.39)$$

$$U = \frac{1}{R} = \frac{k}{L} \quad (9.40)$$

$$P = \frac{kA\Delta T}{L} \quad (9.41)$$

$$A_1 v_1 = A_2 v_2 \quad (9.42)$$

$$P_{\text{net}} = \sigma Ae(T_2^4 - T_1^4) \quad (9.43)$$

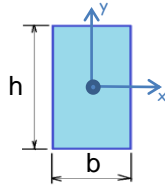
$$k = \frac{PL}{A\Delta T} \quad (9.44)$$

P = rate of heat transfer
Q = thermal energy
A = area of thermal conductivity
U = coefficient of heat conductivity (U-factor)
 ΔT = change in temperature
 Δt = change in time
R = resistance to heat flow (R-value)
k = thermal conductivity
v = velocity
 P_{net} = net power radiated
 $\sigma = 5.6696 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$
e = emissivity constant
L = thickness
 T_1, T_2 = temperature at time 1, time 2

10.0 Section Properties

Moment of Inertia

$$I_{xx} = \frac{bh^3}{12} \quad (10.1)$$



I_{xx} = moment of inertia of a rectangular section about x axis

Complex Shapes Centroid

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i} \quad \text{and} \quad \bar{y} = \frac{\sum y_i A_i}{\sum A_i} \quad (10.2)$$

\bar{x} = x-distance to the centroid

\bar{y} = y-distance to the centroid

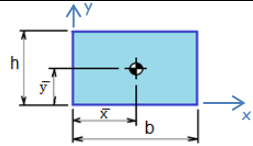
x_i = x distance to centroid of shape i

y_i = y distance to centroid of shape i

A_i = Area of shape i

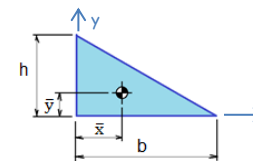
Rectangle Centroid

$$\bar{x} = \frac{b}{2} \quad \text{and} \quad \bar{y} = \frac{h}{2} \quad (10.3)$$



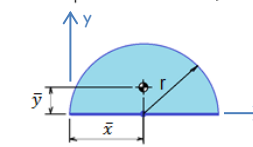
Right Triangle Centroid

$$\bar{x} = \frac{b}{3} \quad \text{and} \quad \bar{y} = \frac{h}{3} \quad (10.4)$$



Semi-circle Centroid

$$\bar{x} = r \quad \text{and} \quad \bar{y} = \frac{4r}{3\pi} \quad (10.5)$$



\bar{x} = x-distance to the centroid

\bar{y} = y-distance to the centroid

11.0 Material

Stress (axial)

$$\sigma = \frac{F}{A} \quad (11.1)$$

σ = stress

F = axial force

A = cross-sectional area

Strain (axial)

$$\epsilon = \frac{\delta}{L_0} \quad (11.2)$$

ϵ = strain

L_0 = original length

δ = change in length

Modulus of Elasticity

$$E = \frac{\sigma}{\epsilon} \quad (11.3)$$

$$E = \frac{(F_2 - F_1)L_0}{(\delta_2 - \delta_1)A} \quad (11.4)$$

E = modulus of elasticity

σ = stress

ϵ = strain

A = cross-sectional area

F = axial force

δ = deformation

12.0 Structural Analysis

Beam Formulas

	<p>Reaction $R_A = R_B = \frac{P}{2}$ (12.1)</p> <p>Moment $M_{\max} = \frac{PL}{4}$ (at point of load) (12.2)</p> <p>Deflection $\Delta_{\max} = \frac{PL^3}{48EI}$ (at point of load) (12.3)</p>
	<p>Reaction $R_A = R_B = \frac{\omega L}{2}$ (12.4)</p> <p>Moment $M_{\max} = \frac{\omega L^2}{8}$ (at center) (12.5)</p> <p>Deflection $\Delta_{\max} = \frac{5\omega L^4}{384EI}$ (at center) (12.6)</p>
	<p>Reaction $R_A = R_B = P$ (12.7)</p> <p>Moment $M_{\max} = Pa$ (12.8)</p> <p>Deflection $\Delta_{\max} = \frac{Pa}{24EI}(3L^2 - 4a^2)$ (at center) (12.9)</p>
	<p>Reaction $R_A = \frac{Pb}{L}$ and $R_B = \frac{Pa}{L}$ (12.10)</p> <p>Moment $M_{\max} = \frac{Pab}{L}$ (at Point of Load) (12.11)</p> <p>Deflection $\Delta_{\max} = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI}$ (12.12) (at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a > b$)</p>

Deformation: Axial

$$\delta = \frac{FL_0}{AE} \quad (12.13)$$

δ = deformation

F = axial force

L_0 = original length

A = cross-sectional area

E = modulus of elasticity

Truss Analysis

$$2J = M + R \quad (12.14)$$

J = number of joints

M = number of members

R = number of reaction forces

13.0 Simple Machines

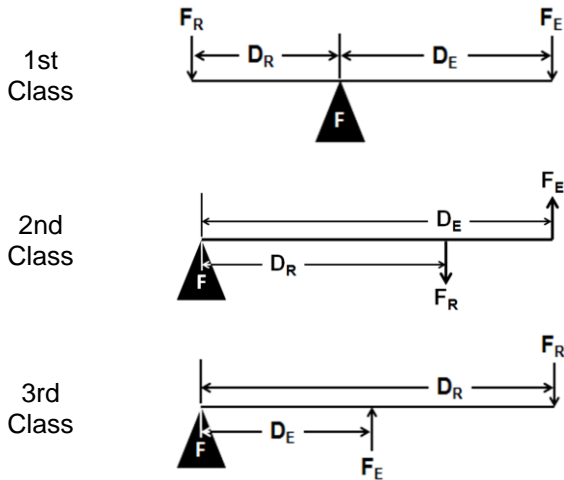
Mechanical Advantage (MA)

$$IMA = \frac{D_E}{D_R} \quad (13.1) \qquad AMA = \frac{F_R}{F_E} \quad (13.2)$$

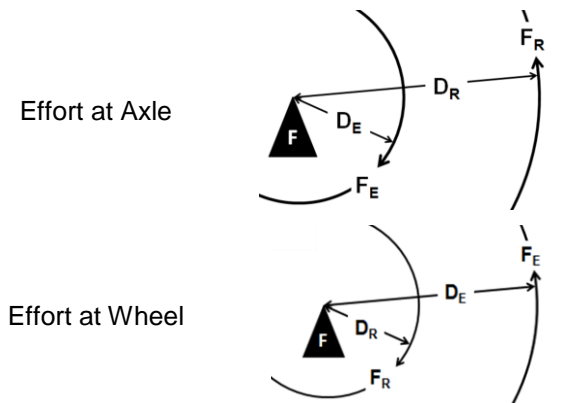
$$\% \text{ Efficiency} = \left(\frac{AMA}{IMA} \right) 100 \quad (13.3)$$

IMA = ideal mechanical advantage
 AMA = actual mechanical advantage
 D_E = effort distance D_R = resistance distance
 F_E = effort force F_R = resistance force

Lever



Wheel and Axle



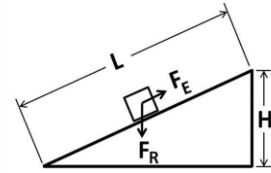
Pulley Systems

IMA = total number of strands of a single string supporting the resistance (13.4)

$$IMA = \frac{D_E \text{ (string pulled)}}{D_R \text{ (resistance lifted)}} \quad (13.5)$$

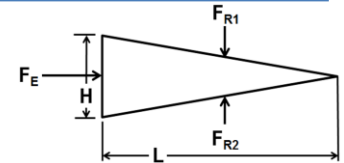
Inclined Plane

$$IMA = \frac{L}{H} \quad (13.6)$$



Wedge

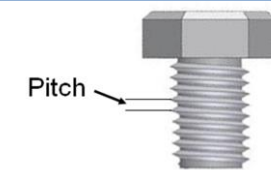
$$IMA = \frac{L}{H} \quad (13.7)$$



Screw

$$IMA = \frac{C}{\text{Pitch}} \quad (13.8)$$

$$\text{Pitch} = \frac{1}{\text{TPI}} \quad (13.9)$$



C = circumference
 r = radius
 Pitch = distance between threads
 TPI = threads per inch

Compound Machines

$$MA_{\text{TOTAL}} = (MA_1) (MA_2) (MA_3) \dots \quad (13.10)$$

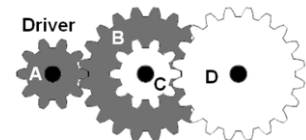
Gears; Sprockets with Chains; and Pulleys with Belts Ratios

$$GR = \frac{N_{\text{out}}}{N_{\text{in}}} = \frac{d_{\text{out}}}{d_{\text{in}}} = \frac{\omega_{\text{in}}}{\omega_{\text{out}}} = \frac{\tau_{\text{out}}}{\tau_{\text{in}}} \quad (13.11)$$

$$\frac{d_{\text{out}}}{d_{\text{in}}} = \frac{\omega_{\text{in}}}{\omega_{\text{out}}} = \frac{\tau_{\text{out}}}{\tau_{\text{in}}} \text{ (pulleys)} \quad (13.12)$$

Compound Gears

$$GR_{\text{TOTAL}} = \left(\frac{B}{A} \right) \left(\frac{D}{C} \right) \quad (13.13)$$



GR = gear ratio
 ω_{in} = angular velocity - driver
 ω_{out} = angular velocity - driven
 N_{in} = number of teeth - driver
 N_{out} = number of teeth - driven
 d_{in} = diameter - driver
 d_{out} = diameter - driven
 τ_{in} = torque - driver
 τ_{out} = torque - driven

18.0 Hazen-Williams Constants

Pipe Material	Typical Range	Clean, New Pipe	Typical Design Value
Cast Iron and Wrought Iron	80 - 150	130	100
Copper, Glass or Brass	120 - 150	140	130
Cement lined Steel or Iron		150	140
Plastic PVC or ABS	120 - 150	140	130
Steel, welded and seamless or interior riveted	80-150	140	100

19.0 Equivalent Length of (Generic) Fittings

Screwed Fittings		Pipe Size										
		1/4	3/8	1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	4
Elbows	Regular 90 degree	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
	Long radius 90 degree	1.5	2.0	2.2	2.3	2.7	3.2	3.4	3.6	3.6	4.0	4.6
	Regular 45 degree	0.3	0.5	0.7	0.9	1.3	1.7	2.1	2.7	3.2	4.0	5.5
Tees	Line Flow	0.8	1.2	1.7	2.4	3.2	4.6	5.6	7.7	9.3	12.0	17.0
	Branch Flow	2.4	3.5	4.2	5.3	6.6	8.7	9.9	12.0	13.0	17.0	21.0
Return Bends	Regular 180 degree	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
Valves	Globe	21.0	22.0	22.0	24.0	29.0	37.0	42.0	54.0	62.0	79.0	110.0
	Gate	0.3	0.5	0.6	0.7	0.8	1.1	1.2	1.5	1.7	1.9	2.5
	Angle	12.8	15.0	15.0	15.0	17.0	18.0	18.0	18.0	18.0	18.0	18.0
	Swing Check	7.2	7.3	8.0	8.8	11.0	13.0	15.0	19.0	22.0	27.0	38.0
Strainer		4.6	5.0	6.6	7.7	18.0	20.0	27.0	29.0	34.0	42.0	

Flanged Fittings		Pipe Size																
		1/2	3/4	1	1 1/4	1 1/2	2	2 1/2	3	4	5	6	8	10	12	14	16	18
Elbows	Regular 90 degree	0.9	1.2	1.6	2.1	2.4	3.1	3.6	4.4	5.9	7.3	8.9	12.0	14.0	17.0	18.0	21.0	23.0
	Long radius 90 degree	1.1	1.3	1.6	2.0	2.3	2.7	2.7	3.4	4.2	5.0	5.7	7.0	8.0	9.0	9.4	10.0	11.0
	Regular 45 degree	0.5	0.6	0.8	1.1	1.3	1.7	2.0	2.5	3.5	4.5	5.6	7.7	9.0	11.0	13.0	15.0	16.0
Tees	Line Flow	0.7	0.8	1.0	1.3	1.5	1.8	1.9	2.2	2.8	3.3	3.8	4.7	5.2	6.0	6.4	7.2	7.6
	Branch Flow	2.0	2.6	3.3	4.4	5.2	6.6	7.5	9.4	12.0	15.0	18.0	24.0	30.0	34.0	37.0	43.0	47.0
Return Bends	Regular 180 degree	0.9	1.2	1.6	2.1	2.4	3.1	3.6	4.4	5.9	7.3	8.9	12.0	14.0	17.0	18.0	21.0	23.0
	Long radius 180 degree	1.1	1.3	1.6	2.0	2.3	2.7	2.9	3.4	4.2	5.0	5.7	7.0	8.0	9.0	9.4	10.0	11.0
Valves	Globe	38.0	40.0	45.0	54.0	59.0	70.0	77.0	94.0	120.0	150.0	190.0	260.0	310.0	390.0			
	Gate						2.6	2.7	2.8	2.9	3.1	3.2	3.2	3.2	3.2	3.2	3.2	3.2
	Angle	15.0	15.0	17.0	18.0	18.0	21.0	22.0	285.0	38.0	50.0	63.0	90.0	120.0	140.0	160.0	190.0	210.0
	Swing Check	3.8	5.3	7.2	10.0	12.0	17.0	21.0	27.0	38.0	50.0	63.0	90.0	120.0	140.0			

20.0 555 Timer Design

$$T = 0.693 (R_A + 2R_B)C \quad (20.1)$$

$$f = \frac{1}{T} \quad (20.2)$$

$$\text{duty-cycle} = \frac{(R_A + R_B)}{(R_A + 2R_B)} \cdot 100\% \quad (20.3)$$

T = period

f = frequency

R_A = resistance A

R_B = resistance B

C = capacitance

21.0 Boolean Algebra

Boolean Theorems

$$X \cdot 0 = 0 \quad (21.1)$$

$$X \cdot 1 = X \quad (21.2)$$

$$X \cdot X = X \quad (21.3)$$

$$X \cdot \bar{X} = 0 \quad (21.4)$$

$$X + 0 = X \quad (21.5)$$

$$X + 1 = 1 \quad (21.6)$$

$$X + X = X \quad (21.7)$$

$$X + \bar{X} = 1 \quad (21.8)$$

$$\bar{\bar{X}} = X \quad (21.9)$$

Commutative Law

$$X \cdot Y = Y \cdot X \quad (21.10)$$

$$X + Y = Y + X \quad (21.11)$$

Associative Law

$$X(YZ) = (XY)Z \quad (21.12)$$

$$X + (Y + Z) = (X + Y) + Z \quad (21.13)$$

Distributive Law

$$X(Y+Z) = XY + XZ \quad (21.14)$$

$$(X+Y)(W+Z) = XW+XZ+YW+YZ \quad (21.15)$$

Consensus Theorems

$$X + \bar{X}Y = X + Y \quad (21.16)$$

$$X + \bar{X}\bar{Y} = X + \bar{Y} \quad (21.17)$$

$$\bar{X} + XY = \bar{X} + Y \quad (21.18)$$

$$\bar{X} + X\bar{Y} = \bar{X} + \bar{Y} \quad (21.19)$$

DeMorgan's Theorems

$$\overline{XY} = \bar{X} + \bar{Y} \quad (21.20)$$

$$\overline{X+Y} = \bar{X} \cdot \bar{Y} \quad (21.21)$$

22.0 Speeds and Feeds

$$N = \frac{CS \left(12 \frac{\text{in.}}{\text{ft}} \right)}{\pi d} \quad (22.1)$$

$$f_m = f_t \cdot n_t \cdot N \quad (22.2)$$

Plunge Rate = $\frac{1}{2} \cdot f_m$

N = spindle speed (rpm)

CS = cutting speed (in./min)

d = diameter (in.)

f_m = feed rate (in./min)

f_t = feed (in./tooth/rev)

n_t = number of teeth

23.0 Aerospace

Forces of Flight

$$C_D = \frac{2D}{\rho v^2 A} \quad (23.1)$$

$$R_e = \frac{\rho v l}{\mu} \quad (23.2)$$

$$C_L = \frac{2L}{\rho v^2 A} \quad (23.3)$$

$$M = Fd \quad (23.4)$$

C_L = coefficient of lift
 C_D = coefficient of drag
 L = lift
 D = drag
 A = wing area
 ρ = density
 R_e = Reynolds number
 v = velocity
 l = length of fluid travel
 μ = fluid viscosity
 F = force
 m = mass
 g = acceleration due to gravity
 M = moment
 d = moment arm (distance from datum perpendicular to F)

Propulsion

$$F_N = W(v_j - v_o) \quad (23.5)$$

$$I = F_{ave} \Delta t \quad (23.6)$$

$$F_{net} = F_{avg} - F_g \quad (23.7)$$

$$a = \frac{v_f}{\Delta t} \quad (23.8)$$

F_N = net thrust
 W = air mass flow
 v_o = flight velocity
 v_j = jet velocity
 I = total impulse
 F_{ave} = average thrust force
 Δt = change in time (thrust duration)
 F_{net} = net force
 F_{avg} = average force
 F_g = force of gravity
 v_f = final velocity
 a = acceleration
 Δt = change in time (thrust duration)

NOTE: F_{ave} and F_{avg} are easily confused.

Energy

$$K = \frac{1}{2} m v^2 \quad (23.9)$$

$$U = \frac{-GMm}{R} \quad (23.10)$$

$$E = U + K = -\frac{GMm}{2R} \quad (23.11)$$

$$G = 6.67 \times 10^{-11} \frac{m^3}{kg \times s^2} \quad (23.12)$$

K = kinetic energy
 m = mass
 v = velocity
 U = gravitational potential energy
 G = universal gravitation constant
 M = mass of central body
 m = mass of orbiting object
 R = Distance center main body to center of orbiting object
 E = Total Energy of an orbit

Orbital Mechanics

$$e = \sqrt{1 - \frac{b^2}{a^2}} \quad (23.13)$$

$$T = 2\pi \frac{a^{\frac{3}{2}}}{\sqrt{\mu}} = 2\pi \frac{a^{\frac{3}{2}}}{\sqrt{GM}} \quad (23.14)$$

$$F = \frac{GMm}{r^2} \quad (23.15)$$

e = eccentricity
 b = semi-minor axis
 a = semi-major axis
 T = orbital period
 a = semi-major axis
 μ = gravitational parameter
 F = force of gravity between two bodies
 G = universal gravitation constant
 M = mass of central body
 m = mass of orbiting object
 r = distance between center of two objects

Bernoulli's Law

$$\left(P_s + \frac{\rho v^2}{2}\right)_1 = \left(P_s + \frac{\rho v^2}{2}\right)_2 \quad (23.16)$$

P_s = static pressure
 v = velocity
 ρ = density

Atmosphere Parameters

$$T = 15.04 - 0.00649h \quad (23.17)$$

$$p = 101.29 \left[\frac{(T + 273.1)}{288.08} \right]^{5.256} \quad (23.18)$$

$$\rho = \frac{p}{0.2869(T + 273.1)} \quad (23.19)$$

T = temperature
 h = height
 p = pressure
 ρ = density